Figure "8" gravitational-wave antenna using a superconducting-core coaxial cable: Continuity equation and its superluminal consequences

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In our Prague paper [1], we predicted that the Cooper pairs will respond to a gravitational (GR) wave *differently* from the ions of the ionic lattice of a superconductor (SC), because there exists a global rigidity of the wavefunction of the former which is absent in the latter. This results in a *differential* motion between the pairs and the lattice. Therefore a supercurrent will be induced by the gravitational wave. This induced supercurrent leads in turn to a charge-accumulation effect at the boundaries of the SC where the supercurrent terminates. The existence of the charge-accumulation effect follows from the continuity equation.

Furthermore, we predicted that the supercurrents induced by an incident GR wave will be *superluminal*. This is one of the most counter-intuitive results of our Prague paper. We calculated that extremely fast (i.e., "instantaneously superluminal") electronic mass motions will be induced by a GR wave incident on a square SC plate. For example, a Cooper pair could disappear from point A at the edge of the first quadrant of the plate, and then *instantaneously* reappear at point B at the edge of the second quadrant of the plate, even when the points A and B are extremely far away from each other. The resulting huge mass supercurrents within the SC plate induced by the GR wave could then lead to a mirror-like reflection of the wave. We shall show here how such superluminal supercurrents can arise, starting from the continuity equation.

Consider the SC-core coaxial cable that is bent into the shape of a figure "8" shown in Figure 1. The normal metallic parts of the cable, indicated in orange (denoting copper) are transparent to gravitational radiation, which is incident from the left. As this radiation penetrates into the cable it will encounter its SC core, indicated in blue (denoting superconducting aluminum). The ions of the ionic lattice of the cable will undergo inhomogeneous free fall with a *local* acceleration due to tidal gravitational forces which are denoted by $\mathbf{g}$ or $-\mathbf{g}$ in Figure 1, depending on the location of the ions [3].

However, the *non-localizable* Cooper pairs, which have a *globally* coherent quantum phase everywhere, will not undergo free fall along with the *local* motion of the ions. Therefore there will result a supercurrent which circulates around

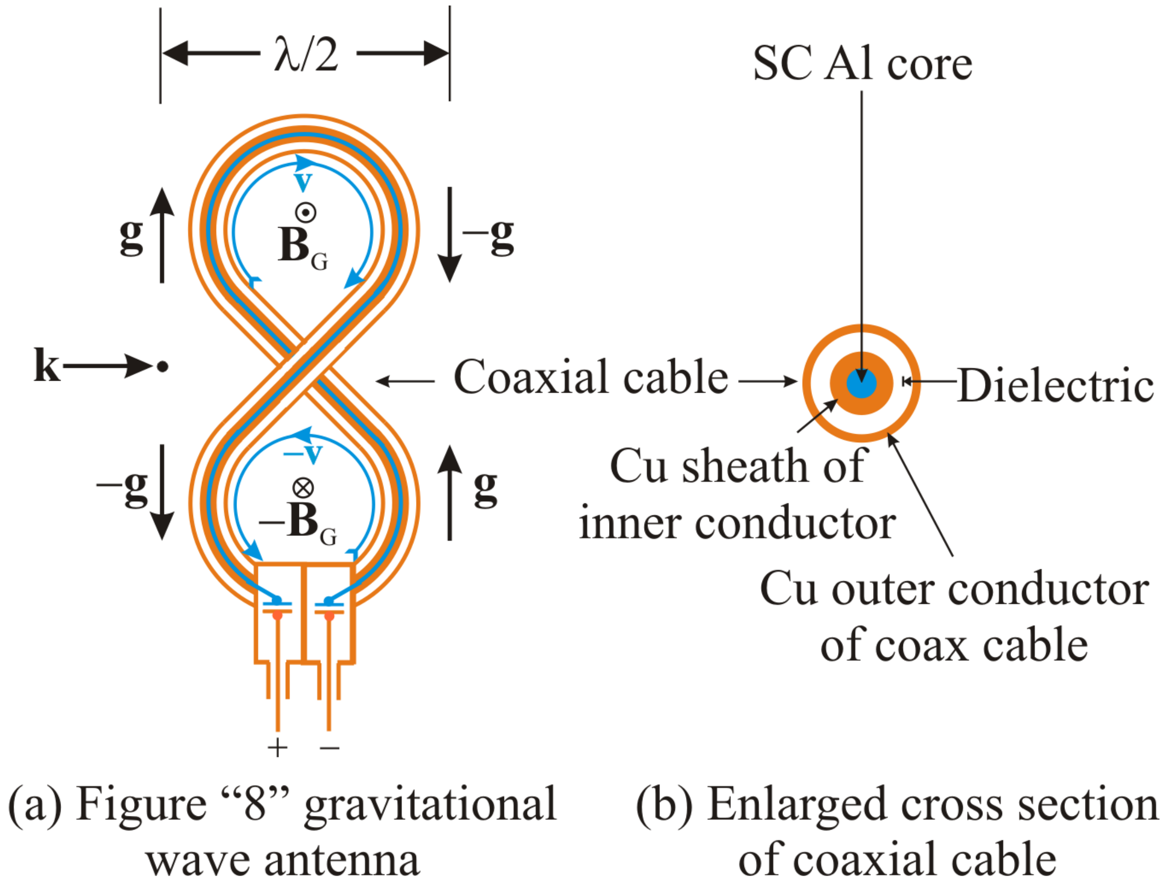

Figure 1: (a) Snapshot at $t = 0$ of a gravitational plane wave which is incident with wavevector $\mathbf{k}$ from the left upon a superconducting (SC)-core coaxial cable that is bent into the shape of a figure "8." The instantaneous local accelerations at $t = 0$ due to gravity are denoted by $\mathbf{g}$ and $-\mathbf{g}$ [2]. The width of the antenna is approximately $\lambda/2$, where λ is the wavelength of the gravitational wave. The superfluid velocity of the Cooper pairs is denoted by $\mathbf{v}$ and the gravito-magnetic field by $\mathbf{B}_{\mathrm{G}}$. (b) Enlarged cross-sectional view of the coaxial cable. *Blue* denotes the SC aluminum core of the cable; *orange* denotes the normal copper sheath surrounding the SC core of the inner conductor, and also denotes the outer conductor of the cable, as well as the two Faraday cages which contain the two bimetallic capacitors shown at the bottom of the figure "8" in (a).

the figure "8" in a clockwise sense in the upper half of the figure "8", but will circulate around in an anti-clockwise sense in the lower half. This supercurrent will be terminated by the blue plates of the bimetallic capacitors inside the electronic boxes are the bottom of the figure "8", and therefore charges will accumulate at these blue plates. Oppositely signed charges + and − will then be induced on the two orange plates of these bimetallic capacitors. The resulting voltage signals can then be fed through normal SMA cables, and detected after amplification using an oscilloscope or spectrum analyzer.

The conversion efficiency of the incident gravitational wave power into the electrical signal power depends on the impedance matching conditions of the figure "8" antenna. Since the SC has zero impedance (no losses), when the figure "8" is short-circuited such that the two blue plates of the bimetallic capacitors are connected to each other by means of a SC short, one expects the antenna to behave like a mirror, and reflect or scatter the incident GR wave efficiently. Hence one expects that it should be possible to impedance match efficiently the antenna to free space in principle.

If so, by reciprocity, the antenna can serve both as a receiver and as a generator of GR waves, with an equal efficiency for the generation as for the reception of GR waves. This implies the possibility of a Hertz-like experiment, in which two coplanar figure "8" antennas are placed side by side, their axes being vertically oriented, with one antenna as the transmitter and the other as the receiver of microwave-frequency GR waves. This can be done within a single dilution refrigerator sample can. Since the coaxial cables are already electrically shielded from each other by the Faraday cages formed by their outer conductors, any signal communicated from one figure "8" to the other can only arise from GR wave communication. As a control experiment, one can demonstrate that the signal disappears above the transition temperature.

In order to understand the operation of the figure "8" antenna, see Figure 2. In the upper half of Figure 2, we consider the usual case of *luminal* propagation at the usual speed of c/n (where n is the refractive index of the dielectric of the cable) of a charge pulse down a SC coaxial cable with a SC (superconducting) central section of the cable, but in which the SC section has no NSC (nonsuperconducting) sheath surrounding it. (*Blue* denotes SC aluminum; *orange* NSC copper). An incident pulse of charge $Q(t)$ is incident from the left and crosses the small gaps between the NSC and SC sections of the cable, because these gaps form high-capacitance capacitors, which behave like RF shorts for the high frequency pulses like those of $Q(t)$ [4]. Note that the charges and currents will produce *transverse* electric and magnetic fields, which are depicted by the solid and the dashed lines, respectively, in part (b) of Figure 2. Thus the charge pulse $Q(t)$ will propagate luminally down the cable towards the right, exciting the usual TEM mode of a coaxial cable.

In the lower half of this Figure, we consider the unusual case of *superluminal* propagation of the charge pulse $Q(t)$ down a SC-core coaxial cable with the central SC section of the inner conductor intimately surrounded by a sheath of NSC material. There exists a good electrical contact between the SC core and the surrounding NSC sheath, so that Cooper pairs can cross at the point P

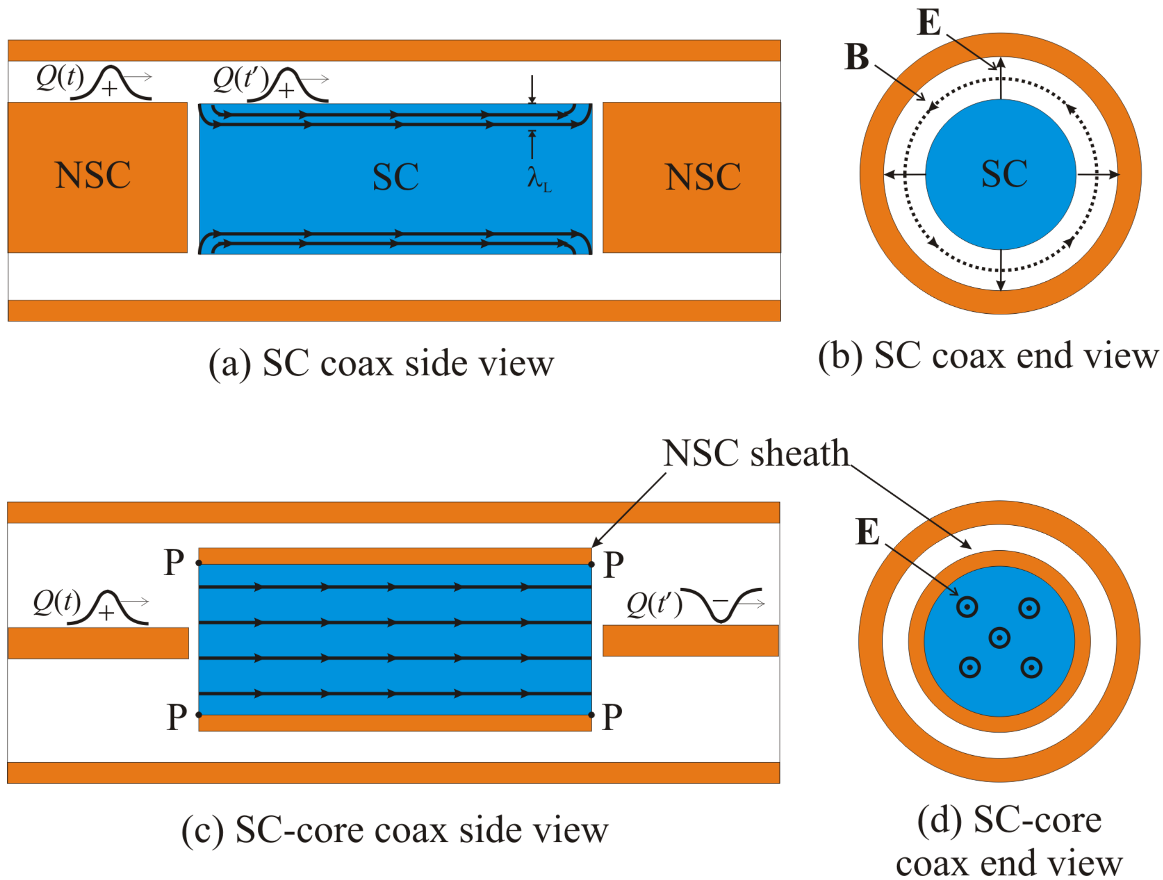

Figure 2: A SC coax *without* a NSC sheath is pictured in the upper half in parts (a) and (b), whereas a SC-core coax *with* a NSC-sheath is pictured in the lower half of in parts (c) and (d). *Luminal* pulse propagation of a charge pulse $Q(t)$ occurs in (a) and (b), whereas *superluminal* pulse propagation occurs in (c) and (d). In parts (b) and (d), the electric field lines $\mathbf{E}$ are denoted by solid lines, and the magnetic field lines $\mathbf{B}$ by dashed lines. In (a), λ_{L} denotes the London penetration depth.

from the SC side into the NSC side of the inner conductor, and break up into normal electrons. These electrons will then diffuse through the NSC sheath to its surface, and travel within a skin depth of the NSC outer surface of the inner conductor. The thickness of the NSC sheath will be chosen to be much thicker than the skin depth, so that the *transverse* electric and magnetic fields of the usual TEM mode of propagation shown in (b) of the upper half of the Figure, will be shorted out. Therefore only the *longitudinal* electric fields shown in part (d) of the lower half of the Figure, will be allowed to propagate down the cable.

Hence at the point P, there will occur a *partitioning* of charges into two types: *Type (i)* charges are electrons that result from a pair-breaking process, in which Cooper pairs cross at P from the SC core into the NSC sheath, and break up into normal electrons that diffuse towards the surface of the sheath; *type (ii)* charges are unbroken Cooper pairs that remain inside the core of the SC cable and propagate along the equally-spaced straight-line trajectories denoted by the solid black horizontal lines in part (c) of Figure 2 straight to the other end of the cable. Type (i) charges produce *transverse* EM fields, and will therefore travel as a *luminal* charge pulse down the cable, but type (ii) charges will be shown below to produce only *longitudinal* electric fields. It will be shown below that type (ii) charges will travel as a *superluminal* charge pulse down the cable.

To see why superluminal propagation is possible in lower half of Figure 2, let us start from the continuity equation for the charged currents associated with the motion of the Cooper pairs, viz.,

$$\nabla \cdot \mathbf{j} + \frac{\partial \rho}{\partial t} = 0 \tag{1}$$

where $\mathbf{j}$ is the Cooper pairs electrical current density, and ρ is their charge density. The physical meaning of this equation is that charge is conserved during the motion of the Cooper pairs. We define a time-dependent superflow velocity field $\mathbf{v}$ of the Cooper pairs through the relationship

$$\mathbf{j} = \rho \mathbf{v} \ , \tag{2}$$

where the charge density ρ is related to the complex order parameter ψ as follows:

$$\rho = q\psi^* \psi \tag{3}$$

where q is the charge of a Cooper pair.

To avoid the enormous Coulomb energies associated with the unbalanced charge densities arising from inhomogeneities in the Cooper pair density inside the SC, one demands that the charge density of the ionic lattice must be *exactly* compensated by the charge density of the Cooper pairs at every point deep inside the bulk of the SC away from the surface [5]. Since we will also assume that the ionic lattice possesses a constant, *homogeneous* density everywhere inside the SC [3], it follows that the Cooper pair charge density ρ must be a constant of the motion, viz.,

$$\rho = q\psi^* \psi = \text{ const.} \tag{4}$$

This is consistent with the fact that the ground BCS state of the SC corresponds to a uniform charge-density state, and the fact that in first-order perturbation theory, the ground state remains unaltered to lowest order by perturbations from all radiation fields. It follows that

$$\frac{\partial \rho}{\partial t} = 0 \ . \tag{5}$$

Therefore

$$\rho \nabla \cdot \mathbf{v} + \frac{\partial \rho}{\partial t} = \rho \nabla \cdot \mathbf{v} = 0 \tag{6}$$

and therefore that

$$\nabla \cdot \mathbf{v} = 0 \ . \tag{7}$$

This implies that the superflow velocity field $\mathbf{v}$ of the Cooper pairs is *incompressible.*

From DeWitt's minimal coupling rule [1], we showed that

$$\mathbf{v} = -\frac{q}{m}\mathbf{A} - \mathbf{h} \ , \tag{8}$$

where q is the charge and m is the mass of the Cooper pair, respectively, $\mathbf{A}$ is the vector potential, and $\mathbf{h}$ is DeWitt's vector potential. The DeWitt (or "radiation") gauge is being assumed here, with

$$\nabla \cdot \mathbf{h} = \nabla \cdot \mathbf{A} = 0 \ . \tag{9}$$

Taking the curl of the superfluid velocity field given by (8), one obtains

$$\nabla \times \mathbf{v} = -\frac{q}{m}\nabla \times \mathbf{A} - \nabla \times \mathbf{h} = -\frac{q}{m}\mathbf{B} - \mathbf{B}_{\mathrm{G}} \ , \tag{10}$$

where $\mathbf{B} = \nabla \times \mathbf{A}$ is the magnetic field, and $\mathbf{B}_{\mathrm{G}} = \nabla \times \mathbf{h}$ is the gravito-magnetic field [1]. Using Stokes's theorem, one finds that

$$\oint_{\Gamma} \mathbf{v} \cdot d\mathbf{l} = \int_{S(\Gamma)} (\nabla \times \mathbf{v}) \cdot d\mathbf{S} = -\frac{q}{m}\Phi - \Phi_{\mathrm{G}} \ , \tag{11}$$

where Γ is a closed curve chosen to be deep in the middle of the core of the SC material in the figure "8" curve of Figure 1, $S(\Gamma)$ is the area enclosed by Γ, Φ is the time-varying magnetic flux enclosed inside $S(\Gamma)$, and Φ_{G} is the time-varying gravito-magnetic flux inside $S(\Gamma)$ induced by the incident GR wave [6]. Due to the presence of the NSC sheath surrounding the SC core of the coax, which shorts out any high-frequency EM fields arising from supercurrents inside the core, and due to the fact that the gravitational flux Φ_{G} is extremely small, it follows that

$$\oint_{\Gamma} \mathbf{v} \cdot d\mathbf{l} = \int_{S(\Gamma)} (\nabla \times \mathbf{v}) \cdot d\mathbf{S} = 0 \tag{12}$$

to a very good approximation. For the straight-line sections of SC coax shown in part (c) of Figure 2, $\mathbf{B}$, and therefore Φ, also vanish due to the suppression of any high-frequency EM fields by the NSC sheath surrounding the SC core. Also, $\mathbf{B}_{\mathrm{G}}$ and Φ_{G} again vanish to a very good approximation due to the weakness of gravito-magnetic fields.

Therefore the superflow velocity field $\mathbf{v}$ for Cooper pairs inside the NSC-sheathed SC-core straight section of the coax shown in (c) and (d) of Figure 2 obeys the two equations

$$\nabla \times \mathbf{v} = 0 \ , \tag{13}$$

$$\nabla \cdot \mathbf{v} = 0 \ . \tag{14}$$

In other words, the superflow of Cooper pairs deep inside the middle of the NSC-sheathed SC core is both *irrotational* and *incompressible.* It follows from (13) that a solution exists of the form

$$\mathbf{v} = \boldsymbol{\nabla}\varphi \tag{15}$$

for some potential function φ, and therefore from (14) that

$$\boldsymbol{\nabla}^2\varphi = 0 \ . \tag{16}$$

Thus φ obeys Laplace's equation, i.e., the Cooper-pair superflow will be *streamline* flow. In the special case of the laminar superflow within a straight pipe of constant cross section, the streamlines are the straight lines indicated in part (c) of Figure 2, which satisfy the 1D solution of Laplace's equation, viz.,

$$\varphi(x, y, z, t) = C(t)x \text{ where } C(t) \text{ is constant independent of } x, y, z, \tag{17}$$

where the x direction has been chosen to coincide with the direction of the superflow shown in Figure 2, part (c). The superflow may be *time-dependent* due to the time variations of the incident charge pulse $Q(t)$; nevertheless, there will exist *instantaneous* streamline solutions *everywhere* having the form given by (17).

We shall call the resulting kind of superluminal effect, "instantaneous superluminality": The charge induced by the incident GR wave in the figure "8" antenna squirts out instantaneously from the two ends of the SC-core coax, as indicated by the + and − signs at the bottom of the figure "8" antenna in part (a) of Figure 1. Likewise, in order for charge to be conserved, the polarity of the transmitted charge pulse on the right hand side of the coax cable in part (c) of Figure 2) must be reversed in sign with respect to the incident charge pulse.

It is the exponential suppression of the *transverse* EM degrees of freedom of the SC-core coax cable by the NSC sheath that forces the Cooper pairs to develop *longitudinal,* Coulomb fields internally within the SC core in response to the incident charge pulse $Q(t)$ in Figure 2, part (c). This kind of "instantaneous superluminality" effect associated with Coulomb fields is highly counterintuitive. Therefore it is necessary that we must first check that this effect really exists in the coiled five-meter SC-core coax cable experiment, but also in the simple thought-experiment to be presented below.

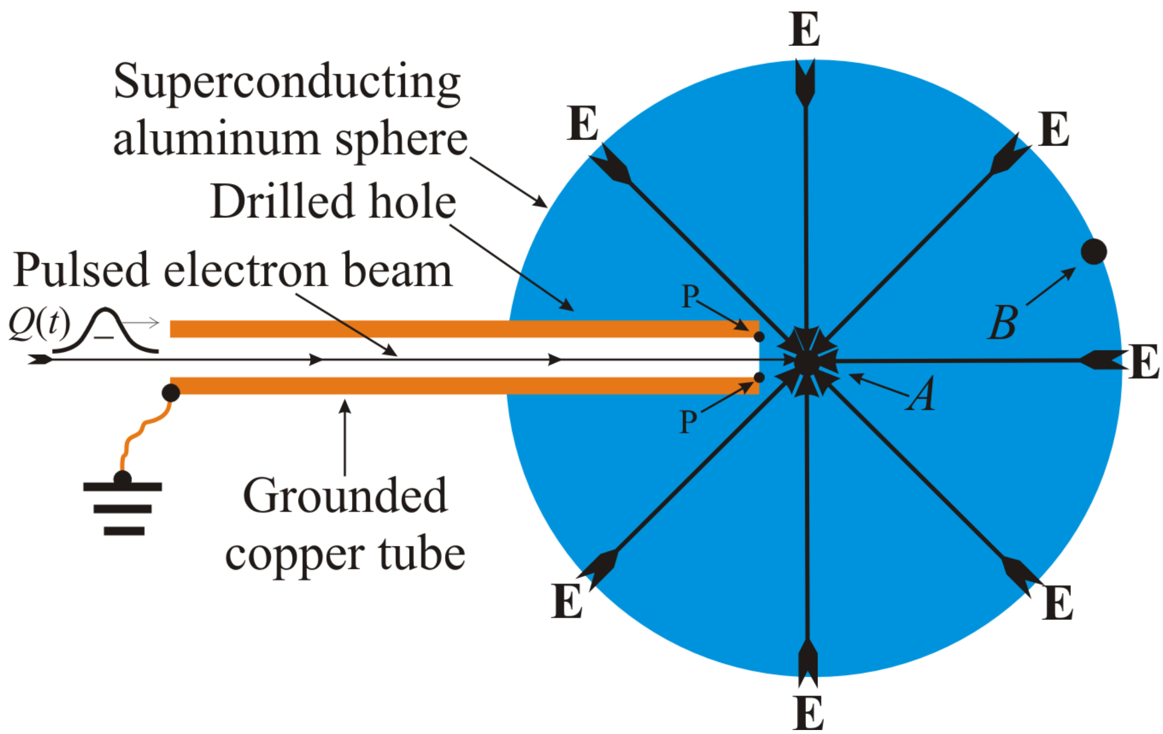

Figure 3: A pulsed electron beam enters through a grounded copper tube (in *orange*) inserted into a drilled hole that ends just before the center (point A) of a superconducting aluminum sphere (in *blue*). The beam is stopped by the aluminum, and its charge is deposited at point A. The deposited charge produces radial lines of electric field $\mathbf{E}$ inside the sphere. How quickly does the deposited charge at point A re-appear at point B as charge on the surface of the sphere?

1 Appendix: Simple model for demonstrating the possibility of "instantaneous superluminality" within a superconducting body

Here we shall show using a simple thought experiment (see Figure 3) how "instantaneous superluminality" in a SC body follows from the continuity equation (1), London's first equation, and Maxwell's first equation.

In Figure 3, a pulsed electron beam enters through a long, hollow non-superconducting copper tube, which is grounded so that it forms a Faraday cage that shields the electrons in the beam. This tube is inserted into a small hole drilled to the center of a SC aluminum sphere, in such a way that the copper tube makes an intimate electrical contact with the aluminum walls of the drilled hole. Thus the copper tube forms a tightly fitting sleeve inside the hole in the aluminum sphere. Hence the electrons in the beam cannot "see" the

Cooper pairs in the SC sphere, nor can the Cooper pairs in the sphere "see" the electrons in the beam, until the beam strikes the aluminum just in front of the center of the sphere at point A, at which point the electrons come to a sudden halt. In this way, electrons will be deposited at point A.

The Coulomb repulsion between the charges thus deposited at A will drive them towards the surface of the sphere. However, there are in principle two different ways by which these charges can reach the surface: They can either travel as *surface* currents along the inner surface of the copper tube, or they can travel as *volume* currents within the body of the aluminum sphere. In the latter case, the charges can be driven towards the surface by a radial Coulomb field, which are indicated by the radial lines of electric field $\mathbf{E}$ in Figure 3. The radial $\mathbf{E}$ field can produce supercurrents $\mathbf{j}$ of Cooper pairs within the volume of the aluminum sphere that can flow out radially towards the surface of the sphere. In the former case, the Cooper pairs produced by the charge deposition can flow towards point P of Figure 3 at the junction between the NSC copper tube and the SC sphere that is closest to the center at point A, where they can undergo a pair-breaking process that converts them back into unpaired electrons that diffuse towards the inner surface of the copper tube. In this way, the deposited charge from the electron beam at point A can in principle escape as *surface* currents, and thus avoid flowing as *volume* supercurrents within the interior of the SC body towards its surface.

At the point P, there will again occur a *partitioning* of charges into two types: *Type (i)* charges are electrons that result from a pair-breaking process occurring at P, in which Cooper pairs cross over from the SC aluminum sphere into the NSC copper tube, and will thus break up into normal electrons that diffuse towards the inner surface of the tube; *type (ii)* charges are unbroken Cooper pairs that remain inside the volume of the SC sphere, and propagate along radial trajectories towards the surface of the sphere. Again, type (i) charges can in principle produce *transverse* EM fields, and will therefore travel as a *luminal* charge pulse propagating within the copper tube, whereas type (ii) charges will be shown below to produce only *longitudinal* electric fields associated with a *superluminal* charge pulse propagating within the aluminum sphere.

However, if the inner diameter of the copper tube is chosen to be small, there will exist a high cutoff frequency of the fundamental TM_{01} mode of propagation of EM waves within the tube. If the spectrum of frequencies in the incident Gaussian charge pulse lies well below this cutoff frequency, the luminal propagation process by type (i) charges along the inner surface of the copper tube will be exponentially suppressed in favor of the superluminal propagation process by type (ii) charges within the volume of the aluminum sphere. The electrons in the incoming electron beam, which can have a very short wavelength, are not affected by this waveguide-type cutoff for EM waves.

Furthermore, charges originating from point A can only reach the ground at the far left end of the copper tube by traveling as luminal type (i) charges along the outer surface of the section of the copper tube that sticks out from the surface of the aluminum sphere in Figure 3. Thus there could exist a considerable

dwell time on the surface of the aluminum sphere for the superluminal type (ii) charges, which could appear instantaneously on the surface of the sphere, but before these charges could leak into the ground, they would have to travel as luminal type (i) charges along the outer surface of an *arbitarily long*, exposed section of the copper tube, which extends all the way from the surface of the sphere to the tube's far left end, where the connection to ground is made.

To start the analysis of the propagation of the type (ii) charges, let us assume that London's first equation holds inside the volume of the SC aluminum sphere, so that everywhere inside the sphere

$$\mathbf{j} = -\Lambda \mathbf{A} \tag{18}$$

where Λ is London's constant, and $\mathbf{A}$ is the vector potential inside the body, which obeys the London (or radiation) gauge

$$\nabla \cdot \mathbf{A} = 0 \ . \tag{19}$$

Therefore the electric field inside the SC is related to the vector potential by

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} \ . \tag{20}$$

Taking the time derivative of the continuity equation (1), one obtains

$$\nabla \cdot \left(\frac{\partial \mathbf{j}}{\partial t} \right) + \frac{\partial^2 \rho}{\partial t^2} = \nabla \cdot \left(-\Lambda \frac{\partial \mathbf{A}}{\partial t} \right) + \frac{\partial^2 \rho}{\partial t^2} = \nabla \cdot (\Lambda \mathbf{E}) + \frac{\partial^2 \rho}{\partial t^2} = 0 \ . \tag{21}$$

Assuming that the superconductor is a homogeneous body, so that Λ is independent of position inside the SC, one concludes that

$$\nabla \cdot (\Lambda \mathbf{E}) + \frac{\partial^2 \rho}{\partial t^2} = \Lambda \nabla \cdot \mathbf{E} + \frac{\partial^2 \rho}{\partial t^2} = \frac{\Lambda}{\varepsilon_0} \rho + \frac{\partial^2 \rho}{\partial t^2} = 0 \ , \tag{22}$$

where we have used Maxwell's first equation $\nabla \cdot \mathbf{E} = \rho/\varepsilon_0$. There results a second-order partial differential equation in time

$$\frac{\partial^2 \rho}{\partial t^2} + \frac{\Lambda}{\varepsilon_0} \rho = 0 \tag{23}$$

which can be rewritten in the form of a simple harmonic oscillator equation of motion

$$\frac{\partial^2 \rho}{\partial t^2} + \omega_p^2 \rho = 0 \tag{24}$$

for some natural frequency ω_p which is given by

$$\omega_p = \left(\frac{\Lambda}{\varepsilon_0} \right)^{1/2} \ . \tag{25}$$

The general solution to the PDE (24) is

$$\rho(\mathbf{r}, t) = A(\mathbf{r}) \cos \omega_p t + B(\mathbf{r}) \sin \omega_p t \tag{26}$$

where the coefficients $A(\mathbf{r})$ and $B(\mathbf{r})$ are to be determined by the initial conditions inside the SC sphere. Taking the first partial derivative with respect to time of (26), one finds that

$$\dot{\rho}(\mathbf{r},t) = -\omega_p A(\mathbf{r}) \sin \omega_p t + \omega_p B(\mathbf{r}) \cos \omega_p t \ . \tag{27}$$

Letting the time t approach the distant past by taking the asymptotic limit $t \to -\infty$ in (26) and (27), one determines that at an arbitrary time t in the distant past and at every point $\mathbf{r}$ in the interior of the SC sphere

$$\lim_{t\to-\infty} \rho(\mathbf{r},t) = A(\mathbf{r}) \cos \omega_p t + B(\mathbf{r}) \sin \omega_p t = 0 \ ; \tag{28}$$

$$\lim_{t\to-\infty} \dot{\rho}(\mathbf{r},t) = -\omega_p A(\mathbf{r}) \sin \omega_p t + \omega_p B(\mathbf{r}) \cos \omega_p t = 0. \tag{29}$$

Solving for the coefficients $A(\mathbf{r})$ and $B(\mathbf{r})$, one finds that

$$A(\mathbf{r}) = 0 \text{ for all } \mathbf{r} \ ; \tag{30}$$

$$B(\mathbf{r}) = 0 \text{ for all } \mathbf{r} \ . \tag{31}$$

This is because in the distant past, as one takes the limit $t \to -\infty$, long before the arrival of the main peak of the charge pulse — which we shall assume for simplicity to be a Gaussian pulse whose peak arrives at $t = 0$ — the SC body is electrically neutral everywhere, so that one requires that $\rho = 0$ as $t \to -\infty$ for all $\mathbf{r}$, and also because in the distant past in the same limit $t \to -\infty$, the body's charge *remains* zero everywhere (i.e., it *stays* electrically neutral in the distant past everywhere), so that one also requires that the first derivative of ρ with respect to time will also zero as $t \to -\infty$ for all $\mathbf{r}$. This implies at all later times and for all points $\mathbf{r}$, except for points at the center of the sphere, where the charge of electron beam is being deposited, and except for the points on the surface of the sphere, where the deposited charge can reappear, that

$$\rho(\mathbf{r},t) = A(\mathbf{r}) \cos \omega_p t + B(\mathbf{r}) \sin \omega_p t = 0 \text{ for all } \mathbf{r} \text{ for all times } t > -\infty \tag{32}$$

including when the main peak of the Gaussian pulse arrives at $t = 0$. This implies that no charge can accumulate at any point $\mathbf{r}$ in the interior of the SC sphere at any time t. Physically, this arises from the fact that the lines of superflow of the supercurrent $\mathbf{j}$, which form streamlines, cannot terminate at any point $\mathbf{r}$ within the interior of the SC body. Therefore no charge can accumulate at any interior point $\mathbf{r}$ at any time t, and the charge density ρ at $\mathbf{r}$ will remain zero for all t. In other words, the charge of the ionic lattice will always be *exactly* compensated by the charge of the Cooper pairs at all interior points within the body.

Hence the only places in the SC body where charge can accumulate, and therefore where the charge density can change with time, is either at the center of the sphere, where the charge from the incoming Gaussian pulse of electrons is being deposited, or at points on the surface of the sphere where this deposited charge re-appears in just such way that the *total* charge of the entire system

is always *exactly* conserved. This implies that the disappearance of a given electron at point A is always accompanied by its *simultaneous* reappearance at an arbitrarily far-away point B on the surface at *exactly* the same instant of time. Otherwise, the principle of charge conservation would be violated.

We shall call this counter-intuitive effect "instantaneous superluminality within a SC body." Note that this superluminal effect does not violate relativistic causality because the incident charge pulse has an analytic waveform, e.g., a Gaussian, with a finite bandwidth (i.e., with frequencies less than the BCS gap). There exists no discontinuous "front" within the Gaussian waveform, before which the waveform is *exactly* zero. Such a "front" would contain infinitely high frequency components that would exceed the BCS gap frequency, and thus destroy the superconductivity of the sphere. Thus this "instantaneously superluminal" effect has similarities with that of a Gaussian wavepacket tunneling through a tunnel barrier in quantum mechanics, whose early analytic tail contains all the information needed to reconstruct the entire transmitted wave packet, including its peak, earlier in time *before* the incident peak could have arrived at a detector traveling at the speed of light [7].

What is the London constant Λ? Recall that DeWitt's minimal coupling rule (8), when one sets $\mathbf{h} = \mathbf{0}$ everywhere, leads to the following expression for the superfluid velocity:

$$\mathbf{v} = -\frac{q}{m}\mathbf{A} \ . \tag{33}$$

The supercurrent density $\mathbf{j}$ is then related to $\mathbf{v}$ by

$$\mathbf{j} = \rho\mathbf{v} = nq\mathbf{v} = -\frac{nq^2}{m}\mathbf{A} \ = -\Lambda \ \mathbf{A}, \tag{34}$$

where n is the number density of Cooper pairs in the SC and $\rho = nq$ is their charge density. It follows that London's constant is given by

$$\Lambda = \frac{nq^2}{m} \tag{35}$$

and therefore that the natural frequency

$$\omega_p = \left(\frac{\Lambda}{\varepsilon_0}\right)^{1/2} \ = \left(\frac{nq^2}{m\varepsilon_0}\right)^{1/2} \tag{36}$$

is simply the SC plasma frequency. However, the plasma frequency ω_p lies in the UV part of the EM spectrum, which is far higher in frequency than the BCS gap frequency. It should be kept in mind that the process of pair-breaking at such high UV frequencies would prevent any *real* plasma excitations within the SC body from occurring at ω_p. Nevertheless, *virtual* excitations of the SC plasma at frequencies lower than the BCS gap frequency, for example, within the RF part of the EM spectrum, can still occur inside the SC, provided that the spectrum of the charge pulse incident on the SC sphere in Figure 3 has an upper frequency cutoff which lies well below the BCS gap frequency.

In order to understand the concept of "virtual plasma excitation," let us consider an analogy of (24) with the SHO equation of motion

$$\frac{d^2x}{dt^2} + \omega_0^2 x = f(t) \tag{37}$$

where x is the displacement of the oscillator, ω_0 is its resonance frequency, and $f(t)$ is a forcing function whose spectrum of frequencies lies well below ω_0. (We assume an oscillator with a unit mass here.) Then the displacement of the oscillator at the low frequencies of the driving force $f(t)$ will "adiabatically follow" this force at each instant of time t, so that to a good approximation, the low-frequency solution is given by

$$x(t) \approx \frac{f(t)}{\omega_0^2} \; . \tag{38}$$

There is neither a time-delay nor an energy loss in the system's response to the driving force, i.e., there is no phase lag nor dissipation in the driven SHO. This can also happen here with the RF-frequency Gaussian charge pulse.

The term "virtual" is being used in a quantum sense, in which a non-dissipative "virtual" transition occurs within a two-level atomic medium when it is driven by an EM wave whose frequency is tuned far below the resonance frequency ω_0 of the atoms. However, these "virtual" transitions, which are driven by the low-frequency EM wave, can still lead to a nontrivial dielectric constant of the medium. "Virtual" transitions are to be distinguished from "real" transitions in a system of two-level atoms, in which *dissipation* of the EM wave occurs during the *absorption* of on-resonance photons by the atoms. Likewise, here "virtual plasma excitations" can lead to *non-dissipative* charge-accumulation effects (or equivalently, charge polarization effects) induced on the SC body. No *real* plasma excitations occur within the SC, but instantaneous Cooper-pair relocations (i.e., "instantaneous superluminality") can occur through these *virtual* interactions.

In connection with the distinction between superluminal and luminal effects, it is important to introduce the conceptual distinction between *longitudinal* and *transverse* supercurrents. Let us define two types of supercurrent as follows:

$$\mathbf{j}_{\|}(\mathbf{r},t) \quad \| \quad \mathbf{E}(\mathbf{r},t) \text{ as "longitudinal supercurrents";} \tag{39}$$

$$\mathbf{j}_{\perp}(\mathbf{r},t) \quad \perp \quad \mathbf{E}(\mathbf{r},t) \text{ as "transverse supercurrents".} \tag{40}$$

For example, longitudinal supercurrents can be produced by the time-varying Coulomb field emanating from the charge being deposited by the pulsed electron beam at the center of the SC sphere, as illustrated in Figure 3. Such time-varying longitudinal supercurrents generated by the Coulomb field do not necessarily produce radiation, and therefore one need not expect the usual retardation effects due to the finite speed of light, which are usually associated with the radiation produced by transverse supercurrents. In fact, it is not true that all time-varying supercurrents must lead in all cases to *retarded* interactions.

As an explicit counter-example, one can point to the longitudinal supercurrents that give rise to the above "instantaneous superluminality" effect.

By contrast, transverse supercurrents will be produced by time-varying EM radiation fields, such as those in the fundamental TEM mode of propagation down a SC coaxial cable, in which the SC inner conductor does not possess any NSC (non-superconducting) sheath surrounding it. Transverse supercurrents will propagate within a London penetration depth λ_{L} of the outer surface of the SC inner conductor, and these supercurrents are perpendicular to the local electric fields which are produced through the action of the time-varying magnetic fields generated by these supercurrents, as illustrated in parts (a) and (b) of Figure 2. Hence time-varying transverse supercurrents can produce EM radiation fields, as well as being produced by them. Therefore, in general, one should expect that the retardation effects that are retarded by the finite speed of light will occur in connection with these transverse supercurrents.

To sum up, *longitudinal* supercurrents can give rise to *superluminal* effects, but *transverse* supercurrents can give rise to *luminal* effects.

References

[1] S.J. Minter, K. Wegter-McNelly, R.Y. Chiao, Physica E **42** (2010) 234–255.

[2] The instantaneous local acceleration due to gravity $\mathbf{g}$ here is identical to the gravito-electric field $\mathbf{E}_{\mathrm{G}}$introduced in [1] in connection with the Maxwell-like equations for gravitational fields.

[3] The question arises of whether or not inhomogeneities in the supercurrent around the figure "8" antenna will result from the tidal fields of the incident GR wave, and therefore whether or not charge accumulation will occur at places along the figure "8" antenna other than at the ends of the cable labelled by the + and − signs located inside the Faraday cages at the bottom of the figure "8" shown in part (a) of Figure 1. Note that in Figure 1, there exists a vertical symmetry axis of the figure "8" antenna. At the point of intersection of this axis with the top of the figure "8", the gravitational field $\mathbf{g}$ will vanish identically by symmetry, since this point of intersection will be located midway between the left and right sides of the antenna where the gravitational fields are $\mathbf{g}$ and $-\mathbf{g}$, respectively. (We shall assume here for the sake of argument that the width of the figure "8" antenna is *exactly* half a wavelength wide, so that $\mathbf{g}(t)$ reverses sign to $-\mathbf{g}(t)$ from the left side to the right side of the antenna for all times t.) Therefore the ionic lattice located at the top of the figure "8" will experience no gravitational force, and the ions located there will have a zero velocity vector at all times with respect to the center of mass of the antenna, i.e., they will always remain motionless with respect to a distant inertial observer. However, at the left and right sides of the antenna where $\mathbf{g} \neq \mathbf{0}$, the ions will experience a gravitational force, and will therefore be set in motion with a nonzero velocity vector either towards or away from at the top of the figure "8". It follows that the lattice in a

normal metal will be either compressed or rarified by the tidal forces of the incident GR wave, and therefore that inhomogeneous strains will start to develop inside the normal metal of the figure "8" antenna, similar to those inside a Weber bar. However, will such strains also develop inside the SC core of the cable? The answer depends on whether the ionic lattice drags the Cooper pairs into co-motion with the ions, or the other way around. In [1], we predicted that the latter is the case, and therefore that no such Weber-bar-like strains should appear within the SC core. Hence the density of Cooper pairs should remain constant around the figure "8" during the passage of the GR wave. "Instantaneous superluminality" in the coiled five-meter SC-core coax cable experiment would be evidence for the correctness of this prediction.

[4] The two capacitors (i.e., the two small gaps in part (a) of Figure 2) behave like shorts for the RF-frequency pulse $Q(t)$, and will therefore transmit the RF pulse from the NSC to the SC sections of the cable through the left gap, and then from the SC to the NSC sections of the cable through the right gap. The RF current within a skin depth of the surface of the inner conductor of the left NSC section will continue across the left gap from the NSC section into the SC section of the cable, due to Maxwell's displacement current, and then flow within a London penetration depth of the surface of the inner conductor of the SC section. Thus the transmitted pulse $Q(t')$ in part (a) of Figure 2 will continue to propagate to the right. The initial neutrality of the SC section of the cable is preserved after the charge pulse $Q(t')$ leaves the left gap, by the development of an exactly compensating oppositely signed charge inside the right plate of the left capacitor. This compensating charge, which arises from Maxwell's displacement current, will remain there. The incident charge pulse $Q(t)$ arriving from the left will deposit all of its charge into the left plate of the left capacitor, and also remain there. Similarly when the transmitted charge pulse $Q(t')$ arrives at the right gap, it will deposit all of its charge into the left plate of the right capacitor, and remain there. An exactly compensating oppositely signed charge will develop inside the right plate of the right capacitor, which again arises from Maxwell's displacement current, and a transmitted charge pulse $Q(t'')$ will then leave the right capacitor and continue to propagate to the right within a skin depth of surface of the inner conductor. Thus the SC coax cable will behave not much differently from a normal coax cable with a single, continuous NSC inner conductor. If the two gaps are very small, the energy required to charge the two capacitors up, which must be taken out from the transmitted pulses, will also be very small, so that there will be very little diminution of the transmitted pulse energy through the SC coax cable.

[5] By points "deep inside" the SC, we mean points many coherence lengths ξ_0 away from the surface of the SC. Also, we assume that we are operating at sufficiently low temperatures so that there are essentially no normal electrons left inside the SC, because they have all condensed into Cooper pairs.

[6] The gravito-magnetic flux Φ_G subtended by the figure "8" antenna will in general be nonzero, since the contributions to the flux contained inside the upper and lower loops of antenna add, and do not cancel, due to symmetry (see Figure 1). The gravito-magnetic field $\mathbf{B}_\mathrm{G}$ is maximum in the middle of these loops because a standing GR wave will in general be set up by reflections of the incident traveling wave from the left and right sides of the figure "8", since they will be separated by half a GR wavelength.

[7] R.Y. Chiao, A.M. Steinberg, in Progress in Optics, Vol. 37, E. Wolf (Ed.), Elsevier, Amsterdam, 1997; R.W. Boyd, D.J. Gauthier, in Progress in Optics, Vol. 43, E. Wolf (Ed.), Elsevier, Amsterdam, 2002.